AN ANALYTICAL PROCEDURE FOR DETERMINATION OF LIFT AND DRAG FORCES FOR BODIES OF REVOLUTION

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TABLE OF CONTENTS

PREFACE

MEANING OF SYMBOLS USED

A	Lamb's Inertia Coefficient Form
a	Body length /2
Area	Maximum body cross-section area, square feet
B	Lamb's Inertia Coefficient Form
C_D	Coefficient of Drag = D/q Area
$C_{D_{min.}}$	Minimum Coefficient of Drag
C_L	Coefficient of Lift = L/q Area
C_N	Coefficient of Normal Lift = $C_D \sin\theta + C_L \cos\theta$
d	Maximum body diameter, feet
D	Drag of body, pounds
F_T	Resultant transverse force on body, pounds
ΔF_T	Transverse force per unit length of body, pounds
K	$f\,(Y \sin 2\alpha)$
K_1	Lamb's Inertia Coefficient = A-1
K_2	Lamb's Inertia Coefficient = B-1

ℓ	Body length, feet
q	Dynamic test pressure = $\rho V^2/2$, pounds per square foot
V	Air velocity, feet per second
Y	Spheroid radius at point on surface, feet
α	Angle between spheroid major axis and tangent to point on surface
$dC_D/d\theta\Big]_{\ell/d}$	Slope of coefficient of drag vs. angle of attack
ρ	Local air density, slugs per cubic foot
θ	Angle of attack for body

LIST OF TABLES

LIST OF FIGURES

FIGURE PAGE

FIGURE PAGE

AN ANALYTICAL PROCEDURE FOR DETERMINATION OF LIFT AND DRAG FORCES FOR BODIES OF REVOLUTION

SUMMARY

Very extensive theoretical studies have been made concerning the flow about bodies of revolution. In comparison, the experimental treatment of the flow about bodies of revolution has been given little consideration. In view of the above, it becomes evident that there exists a need for a practical analytical procedure for the prediction of aerodynamic forces on bodies of revolution.

A study of the theoretical procedures, coupled with an analysis of very recent experimental results, suggested an analytical procedure which successfully predicted the observed drag and lift forces on various shaped bodies of revolution.

Theories for the estimation of aerodynamic properties of bodies of revolution were found to be very cumbersome. Most theories required the construction of elaborate sink and source distributions. The lack of adequate experimental data had prevented the comparison of theoretical and experimental results, which was so valuable to the development of modern airfoil theory, and as a result the characteristics of flow about bodies of revolution were neglected.

INTRODUCTION

Several procedures are available for the analytical prediction of airfoil aerodynamic characteristics, but the same is not true for bodies of revolution. Early theories of hydrodynamics dealt with the flow of a perfect fluid about various shaped bodies of revolution. The inadequacy of the perfect flow theory was that while it did tell something of the velocity distribution in the flow field about the body, it failed to indicate the amount and nature of the lift and drag forces. For the slow speed aircraft, excepting drag, the other aerodynamic forces of the fuselage, nacelle, and other external bodies similar to bodies of revolution could be neglected. However, modern aircraft designs involving high area ratios of body to wing will not permit the neglecting of these forces.

Streamline bodies of revolution yield a flow pattern which resembles in many respects that computed by the classical methods of hydrodynamics for irrotational motion of a non-viscous, incompressible fluid. Observations of the pressure and velocity fields show an almost complete agreement with the classical theory except near the extreme tail and in the wake. Theory yields no resultant force, whereas experiment gives a small resultant force which is not due entirely to skin friction.

Bluff bodies of revolution offer far more of a challenge to the short-comings of the classic theories of hydrodynamics than do the many other contour forms of bodies of revolution. The flow breaks away at the sharp edge of this body, and in theory a singularity is developed which always results in an infinite velocity condition. Attempts have been made to theoretically imitate the observed type of flow velocity, which is not infinite, by assuming surfaces of discontinuity that leave the body at the sharp edges.

In describing the flow about various types of models we must fully appreciate factors which are produced by either turbulence or the limited air stream size; when comparing theory and experiment, these factors must be realized for their ability to cause large departures from the theoretically assumed flow conditions.

Generally speaking, the flow about bodies of revolution has not by any means been completed. It seems very probable that if the theory could be placed in such form as to yield the observed curve of force coefficients without the aid of experimentally determined constants (other than the viscosity and density of the fluid), then the flow could be computed about any body.

Lamb's[1] spheroidal harmonic treatment of the flow about prolate spheroids is considered to be one of the

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[1] Lamb, Horace, Hydrodynamics, Fifth edition; Cambridge: University Press, 1930. pp. 130-132.

most important contributions of all early works that pertained to the field of hydrodynamics. This treatment is quite lengthy, and is applicable only to a spheroid with the major axis parallel to the direction of a fluid flow which is irrotational, non-viscous, and incompressible. La Vier[2] has presented Lamb's work in an expanded and clarified form. Much of the historical background for this thesis was presented in the thesis by La Vier.

In the early twenties, much work was done by the British concerning the design of bodies which were required to offer minimum resistance to motion through a fluid. Pannell and Jones[3] presented a viewpoint that suggested the possibility of the calculation of a coefficient which could be used to predict the drag effect of increasing the length of cylindrical portion in bodies of revolution.

Let C_{c_1} = resistance coefficient for a model with one diameter of cylindrical body.

C_{c_2} = resistance coefficient for a model with two diameters of cylindrical body.

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[2]La Vier, Hurlbut, W. S. "Prediction of Aerodynamic Coefficients of Force and Moment Acting of Bodies of Revolution", A masters thesis, Daniel Guggenheim School of Aeronautics, Georgia Institute of Technology, Atlanta, Georgia, June, 1947. pp. 14-22.

[3]Pannell, J. R., and R. Jones, "Experiments in a Wind Channel on Elongated Bodies of Approximately Streamline Form", Technical Report of the Advisory Committee for Aeronautics, Vol. I, No. 607, 1918-19, pp. 212-264.

k = a ratio (the increase of force per unit area due to increase from C_1 to C_2, to ρv^2)

h_1, h_2 = lengths of C_1 and C_2 respectively

D = diameter

l_1 = cube root of volume of model with C_1

K = Resistant coefficient

R = Model Resistance

ρ = fluid density

v = fluid velocity

Now $$K = R/\rho v^2 \tag{1}$$

$$C_{c_1} = K/l^2 \tag{2}$$

And $$C_{c_2} = \frac{K + \pi D(h_2 - h_1)k}{\left\{l_1^3 + \frac{\pi}{4} D^2 (h_2 - h_1)\right\}^{2/3}} \tag{3}$$

K is calculated from the first formula and the value obtained introduced into the second formula, by means of which the value of k may be found. It may be noted that C and k are non-dimensional, while K is of the dimensions l^2.

Such a calculation was based upon extremely limited experimental results owing to the small number of observations which were available on suitable lengths of models. A realization of this limitation does not permit any practical significance to be attached to the calculation.

Munk[4] developed a theoretical method for determining the distribution of force along the length, applying exactly to ellipsoids and approximately to other body of revolution shapes. This method is dependent upon general considerations of the changes of momentum of the fluid. The shapes of the experimental and theoretical curves are similar on various forms of bodies which were used at that time for airship hulls.

This treatment considered an airship flying in a straight line with velocity V, and with an angle of attack ϕ. In a stationery plane perpendicular to the body axis, the air does not move from the vertical disposition of this plane, and for all practical purposes Munk considers two-dimensional flow to exist about the body, which is moving transversely.

The apparent mass of the two-dimensional flow in a layer of thickness dX, if S is the area of the circle, is $\rho S dX$, since the apparent transverse mass of a circular cylinder, if the flow is two-dimensional, is known to be equal to the mass of the fluid displaced. The transverse momentum in the layer is $\rho SV\sin\phi\, dX$. The rate of change of this is $\rho V\sin\phi\, dX\, dS/dt$.

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[4] Munk, Max M., "The Aerodynamic Forces of Airship Hulls," U. S. National Advisory Committee for Aeronautics Technical Report, Vol. IX, No. 184, 1923. pp. 453-468.

But $$\frac{dS}{dt} = \frac{dS}{dX}\,\frac{dX}{dt} = V\cos\phi\,\frac{dS}{dX} \tag{1}$$

Hence at any element of length dX there is a transverse force on the airship given by

$$\frac{\rho}{2}V^2\sin 2\phi\,\frac{dS}{dX}\,dX = dF$$

(2)

Airships only moderately elongated have resultant forces and a distribution of them differing from those given by formula (2) above. In practice, it may be exact enough to assume the shape of distribution and to modify the transverse forces by constant diminishing factors.

Munk presented reasons for multiplying formula (2) by the factor $k_2 - k_1$, Lamb's inertia coefficients, and the final relation for transverse force per unit length is, where S is the circular cross section area at point X

$$(k_2 - k_1)\frac{\rho}{2}V^2\sin 2\phi\,\frac{dS}{dX} \tag{3}$$

This treatment is essentially based upon the proper selection of inertia coefficients, and much experimental data must be made available in order that the correct coefficient can be selected for the shape distribution which occurs different from the prolate spheroid.

Prandtl[5] advised that the airship hull shape closely resembles the streamlined body, which is the only body of revolution for which the actual flow about the body is very similar to the calculated flow for a similar body in ideal fluid. His analysis further declared that the flow could be mathematically determined for bodies of revolution if along the body axis parallel with the flow was placed any suitable distribution of sources and sinks taking care that the respective total strengths of sources and sinks were the same. For the smoothest possible shape, the sources and sinks should be distributed continuously along the axis.

His method consisted mainly of calculating the streamlines by integrating over a surface indicated by velocity components, and following the meaning of a stream function that the stream function of a flow due to two or more causes is at every point the sum of the stream functions of the several partial flows. By putting the total stream function equal to zero, yields the equation of the body surface, and by putting the total stream function equal to any number of constants, yields the streamlines about the body.

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[5]Prandtl, L., and O. G. Tietjens, "Applied Hydro- and Aero-Mechanics", McGraw-Hill Book Co., Inc., New York, 1934. pp. 137.

From the relations

$$u = \frac{1}{2\pi Y}\frac{\partial \Psi}{\partial Y} \tag{1}$$

And

$$v = -\frac{1}{2\pi Y}\frac{\partial \Psi}{\partial X} \tag{2}$$

Bernoulli's equation may be applied

$$p = p_o + \frac{\rho}{2}\left\{V^2 - (u^2 + v^2)\right\} \tag{3}$$

and the pressure distribution predicted for the body.

Where

u = velocity component parallel to X - axis

v = velocity component parallel to Y - axis

Ψ = stream function

Y = any point in a plane perpendicular to X - axis

p = pressure at point on body surface

p_o = free stream pressure

ρ = stream density

V = stream velocity

Prandtl's report includes sketches of various streamline body pressure distributions, and clearly portrays the close agreement between theory and test results. This theory holds true only when the body axis is in line with the stream flow, and therefore is not applicable for various angles of attack.

Pressure distribution about and resistance of simple quartics fixed in an infinite uniform stream of practically incompressible fluid has been treated by Zahm[6]. He determined the velocity potential Φ and stream function Ψ for the sphere, infinitely long cylinder transverse to flow, elliptic cylinder, prolate spheroid, oblate spheroid, and circular disc normal to the flow. From the velocity potential and the stream function, the velocity component formulas were derived for each quartic. La Vier[7] has clearly defined the important mathematical expressions and relations, and this author will not attempt any further clarification.

For the problem of oblique flow, Zahm conducted an analysis which in essence was to resolve the oblique stream in chosen directions into component streams each having its individual flow value at a given point. Combining the individuals yielded the resultant velocity, whence the pressure is found. This solution would involve a new flow pattern analysis for each oblique flow angle, and would therefore be too cumbersome for practical usage.

A comprehensive comparison of theoretical and experimental determination of the distribution of pressure over a prolate spheroid was conducted by Jones[8].

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[6] Zahm, A. F., "Flow and Drag Formulas for Simple Quartics", U. S. National Advisory Committee for Aeronautics Technical Report, No. 253, 1926. pp. 22.

[7] La Vier, op cit., pp. 3-8.

[8] Jones, R, "The Distribution of Normal Pressures on a Prolate Spheroid", Advisory Committee for Aeronautics Reports and Memoranda, No. 1061, 1926-7. pp. 516-602.

This study resulted from the desire to develop procedures by which airloads might be predicted in the design of airship hulls. Actually, this report is important for a two-fold reason: It has quantitative experimental data compared with theoretically determined pressures and forces; also, the analysis takes into consideration oblique flow, and the mathematical treatment can be handled, even though with difficulty.

Jones utilized Lamb's[9] Spheroidal Harmonics method for mathematically determining an expression that could predict the pressure distribution. This author will not attempt to present an analysis of this procedure, but feels it is sufficient to say that Jones was able to integrate the pressures and he obtained drag and lateral force values which were in good agreement with the experimental values.

In the early thirties it was now evident that some theoretical data was available for the pressure distribution over ellipsoids with axes at angles to the wind and experiments had been made on a prolate spheroid in rectilinear motion. Experimental data were available for special shapes used for airship hulls, and approximate methods had been developed for the computation of the pressure distribution over these hulls.

Approximate methods had been developed for finding

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[9]Lamb, _Loc. cit._

the source and sink distribution giving rise to particular shapes, and the source and sink method was extended by Karman[10] to the case where the body axis was at an angle to the wind. This procedure assumed that the flow is produced by superposing a flow arising from a system of sources and sinks on the parallel flow of velocity U. The system consisted of line sources and sinks of differing strengths, in which the yield per unit length is kept systematically constant over the hull length until the resulting flow or streamline pattern has a stream function $\Psi = 0$. This boundary then approximates the hull contour.

Karmen's entire procedure is extremely cumbersome, and must be considered impractical for design work. A flow pattern must be determined for each angle of attack, and finally the pressure distributions must be integrated to yield the transverse force distribution. For the symmetrical flow case, this procedure would not yield the drag, and the oblique flow case will not yield the normal force.

Upson and Klickoff[11] have further developed Jones's[12]

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[10] von Karman, Theodor, "Calculation of Pressure Distribution on Airship Hulls", U. S. National Advisory Committee for Aeronautics Technical Memoranda, No. 574, July, 1930. pp. 27.

[11] Upson, Ralph H., and W. A. Klikoff, "Application of Practical Hydrodynamics to Airship Design", U. S. National Advisory Committee for Aeronautics Technical Report, No. 405, 1932. pp. 123-132.

[12] Jones, op. cit., pp. 519-526.

theory for the calculation of pressure and force distributions for a prolate spheroid. These two men were seeking a conservative estimate of the shear forces over the hull length of an airship, and they were satisfied to get a relation which was based upon perfect fluid flow. Therefore, their results were not modified to account for the differences between theory and experimental.

They suggested that an equivalent spheroidal shape could be chosen so as to have the same apparent mass properties as any arbitrary body of revolution, and this reasoning led this author to consider a further modification of their theory as the basis for this thesis. La Vier's[13] analysis of Upson and Klickoff's report has clearly presented the necessary relations which lead to the following expression for transverse force per unit length of body

$$\Delta F_T = q \frac{AB}{2} \pi Y \sin 2\alpha \sin 2\theta$$

where $q = \rho/2 \, V^2$

$A = 1 + K_1$

$B = 1 + K_2$

K_1 and K_2 are Lamb's Inertia Coefficients for prolate spheroids (see figure 16).

Y = radius of spheroid at point on spheroid surface.

α = angle between spheroid major axis and

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[13] La Vier, op. cit., pp. 23-25.

tangent to point on spheroid surface (see figure 17).

θ = angle of attack

A considerable study of bodies of revolution has been conducted at the Georgia Institute of Technology. In a report by Bryan[12], very sound data has been accumulated concerning the apparent mass inertia coefficients of various bluff bodies of revolution, and these data have been incorporated in the theoretical analysis of this thesis. Rainey[13] conducted excellent experiments for the determination of aerodynamic characteristics of various shapes of bodies of revolution. These results show excellent agreement when correlated with the small amount of data from previous experiments that were conducted for the same purpose, and his data have been used for correlation with the analytical prediction results in this report. La Vier's[14] report describes an analytical prediction of the drag, lift, and moment characteristics for bodies of revolution. His work lacked experimental data for comparison with the theory,

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[12] Bryan, Colgan H., "Apparent Additional Mass Characteristics of Various Bluff Bodies of Revolution", A Masters thesis, Daniel Guggenheim School of Aeronautics, Georgia Institute of Technology, Atlanta, Georgia, September, 1948. pp. 82-85.

[13] Rainey, Robert W., "Experimental Determination of Aerodynamic Characteristics of Cylindrical and Spheroidal Bodies of Revolution", A Masters thesis, Daniel Guggenheim School of Aeronautics, Georgia Institute of Technology, Atlanta, Georgia, March, 1948. pp. 33-37.

[14] La Vier, op. cit., pp. 26-38.

and must be considered in this light when analyzing the report. Much of his theory is the basis for this report, and by extensive modifications his work has been developed into the theory which appears in this report.

DEVELOPMENT OF THE PROCEDURE

Prediction of Normal Force.

The formula for ΔF_T[15] may be expressed as the theoretical transverse force distribution[16] for the prolate spheroid. A study of this formula for the incremental force per unit length of hull

$$\Delta F_T = q \pi \frac{AB}{2} Y \sin 2\alpha \sin 2\theta \tag{1}$$

indicated that the only variable term which is a function of the fineness ratio, is

$$Y \sin 2\alpha \tag{2}$$

accordingly, dividing the spheroid into increments of length and plotting the variation[17] of the above function with length, the basic pattern is obtained for the theoretical transverse force distributions for prolate spheroids of fineness ratios in the interval $2 \leq l/d \leq 10$.

It was observed that if a mechanical integration were made of the area under Y Sin 2α, then the form of

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[15] Cf. ante., p. 13.

[16] La Vier, op. cit., P. 68, Figure 6.

[17] Ibid., pp. 44-51.

this function became

$$\Sigma \, Y \sin 2\alpha \qquad (3)$$

Now the resultant theoretical transverse force relation was

$$F_T = q \pi \frac{AB}{2} \Sigma Y \sin 2\alpha \sin 2\theta \qquad (4)$$

and the theoretical transverse force coefficient was obtained by

$$C_N = \frac{F_T}{q(\text{AREA})} = \frac{\cancel{q} \pi \frac{AB}{2} \Sigma Y \sin 2\alpha \sin 2\theta}{\cancel{q}(\text{AREA})} \qquad (5)$$

where

Area = body cross-section area

Actually, none of the above relations would yield any value other than zero because they were based upon perfect flow conditions in which there can be no resultant lift force. However, the train of thought permitted the development of relations which would apply to the actual flow case.

As $Y \sin 2\alpha$ was a geometric function of fineness ratio, and could not vary from the original theoretical distribution, it was considered that the resulting spheroid $KY \sin 2\alpha$ distribution for actual flow was an equivalent value where $K = f(Y \sin 2\alpha)$, and Formula (5) now took the form

$$C_N = \frac{\pi \frac{AB}{2} \sum KY \sin 2\alpha \sin 2\theta}{\text{Area}} \qquad (6)$$

Formula (6) was applied to prolate spheroid data[18] in the following manner. For the individual spheroid, Lamb's Inertia Coefficients were obtained from Figure 1; C_N, $\pi/2$, Sin 2θ, and body Area values were placed in the formula, and values were obtained for $\sum KY \sin 2\alpha$ as shown in Figure 2.

Formula (6) was then applied to data[19] for bodies of revolution with bluff, conical, and hemispherical faces. First, from Figure 3[20], the equivalent prolate spheroid was determined for the above respective bodies of revolution, and Lamb's Inertia Coefficients were found in Figure 1, for the equivalent prolate spheroid. Upon the application of C_N, $\pi/2$, Sin 2θ, and the Areas for the respective bodies of revolution to Formula (6), Figure 4 was

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[18] Rainey, op. cit., p. 36, Table IV.

[19] Ibid., pp. 33-35.

[20] Bryan, op. cit. pp. 82-85, Figures 7, 8, 9, 10.

found to illustrate the pattern of the ratios for the equivalent spheroid flow values of $\sum KY \text{ Sin } 2\alpha$ over the $\sum KY \text{ Sin } 2\alpha$ equivalent values for the bodies of revolution.

On applying the above analysis to bodies of revolution in a variety of tests, the normal force coefficient was predicted in the following manner from the relation

$$C_N = \frac{\pi AB}{2} \sum KY \text{ Sin } 2\alpha \text{ Sin } 2\theta \ / \text{ Area}$$

For the given body of revolution, obtain the equivalent spheroid fineness ratio from Figure 3, and then determine from Figure 1 Lamb's Inertia Coefficient Product AB, using the equivalent spheroid fineness ratio. Using the original body fineness ratio, obtain $\sum KY \text{ Sin } 2\alpha$ from Figure 2 for a prolate spheroid. Multiply this latter value by the ratio obtained from Figure 4 for $\sum KY \text{ Sin } 2\alpha$ of the spheroid over $\sum KY \text{ Sin } 2\alpha$ for the original body of revolution, and place the product in the normal lift force coefficient formula. Sin 2θ, $\pi/2$, and the original body cross-sectional are applied, and the complete formula will provide the predicted normal lift coefficient.

Figures 6, 7, 8, and 9 portray the accuracy of predictions.

Prediction of Drag.

Early experiments[21] revealed that for all practical purposes, C_D is not a function of Reynolds Number in either the laminar or turbulent flow ranges for bodies of revolution. This analytical prediction of drag for bodies of revolution was based upon test conditions conducted over a range of Reynolds Number from 214,000 to 2,800,000, and no difficulties were encountered with the prediction method. Experiment indicated that the proportion of total drag due to friction is small, or the variance of C_D with Reynolds Number is not great on either side of the transition range for bodies of revolution.

The initial approach to this problem was to classify the $C_{D_{min.}}$ values for the various bodies of revolution at zero angle of attack. Figure 5 indicates the result of this classification procedure, and the various body shape curves are in excellent agreement with similar results achieved by many sources.

All available published and unpublished data, when plotted with C_D vs. Θ, indicated a close relationship between the slopes $dC_D/d\Theta]_{l/d}$ for fineness ratios common to the various bodies of revolution except the spheroids.

[21] von Mises, Richard, Theory of Flight, New York: McGraw-Hill Book Co., 1945. pp. 99-105.

Further analysis revealed that for each fineness ratio, an equation of the form $C_D = C_{D_{min.}} + dC_D/d\theta]_{l/d}\,\theta$ was applicable, and Table 1 indicates these slope values for the various body shapes.

For the spheroids, earlier data indicated and checked with recent tests, the fact that C_D did not change appreciably for a varying fineness ratio, at zero angle of attack, as indicated by Figure 6[22], until the very low fineness ratios 4 are approached. These latter ratios are in the flat plate range, and high $C_{D_{min.}}$ values can be expected.

Accumulated data indicated for the spheroids that as the fineness ratio increased, the curves[23] for C_D vs. θ yielded equal slopes, for all practical purposes. On the basis of this analysis, an equation of the form $C_D = C_{D_{min.}} + dC_D/d\theta]_{l/d}\,\theta$ was applicable, and Table 1 indicates the value of $dC_D/d\theta]_{l/d}$ which is common to all spheroids investigated.

The following procedure for predicting the drag coefficient for bodies of revolution, is based upon the above analysis.

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[22]Rainey, _op. cit._, p. 50, Figure 12.

[23]_Cf. post._, p. 46, Figure 18.

Given a body of revolution, obtain that body's classified $C_{D_{min.}}$ from Figure 5. Then refer to Table 1, where the $dC_D/d\theta]_{\ell/d}$ value may be found. Apply these factors to the following equation, and obtain the predicted C_D value

$$C_D = C_{D_{min.}} + dC_D/d\theta]_{\ell/d}\,\theta$$

Figures 10, 11, 12, 13, 14, and 15 illustrate the accuracy of this prediction method.

<u>Demonstration of Procedure</u>

A cone face cylindrical body[24] of revolution, Figure 18, was recently tested at sixty-six miles per hour. The test data of coefficients for drag and lift, and the transfer of these lift coefficients perpendicular to the longitudinal axis are presented in Table II. A comparison of predicted normal lift coefficients and drag coefficients is also shown in Table II.

The fineness ratio for this body was $\ell/d = 6$ and reference to Figure 3 gives an equivalent prolate spheroid of

$$\ell/d = 1.87$$

which has the same apparent mass characteristics

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[24]Rainey, <u>op</u>. <u>cit</u>., p. 59, Figure 21.

as the test body of revolution.

For $l/d = 1.87$, and from Figure 1,

$$AB = 2.067$$

or
$$\frac{\pi AB}{2} = 3.24$$

Now, for the test body, $l/d = 6$, Figure 4 yields the

Correction Ratio = 1.382

and Figure 2 gives

$$\Sigma KY \text{ Sin } 2\alpha = .0311a^2$$

for a spheroid of $l/d = 6$, where

$$a = \frac{\text{body length}}{2} = 0.375 \text{ ft.}$$

The corrected ΣKy Sin 2α for the test body is

Correction Ratio (Σ KY Sin 2α)

or
$$1.382\ (\ .0311)\ (\ .375)^2$$

Values of the predicted normal force coefficients for several angles of attack are then found by the relation

$$C_N = \frac{\pi\ AB}{2} \quad \Sigma Ky \text{ Sin } 2\alpha \text{ Sin } 2\theta \ / \text{ Area}$$

or

$$C_N = 3.24\ (1.382)\ (\ .0311)\ (\ .375)^2 \text{Sin2} \ /.01266$$

where .01266 sq. ft. is the maximum body cross-section area. Table II reveals the very close agreement between these predicted lift values and the test values.

As was stated in the drag prediction procedure[25],

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[25]Cf. <u>ante</u>., p. 20.

a value of

$$C_{D_{min.}} = 0.496$$

is found in Figure 5, for the test body of

$$\ell/d = 6$$

For this fineness ratio, Table I expresses

$$dC_D/d\theta\Big]_{\ell/d} = 1.33$$

for the body of revolution with a cone nose. The C_D values of the body, for varying angles of attack, may now be predicted by

$$C_D = C_{D_{min.}} + \left(dC_D/d\theta \right)_{\ell/d} \theta$$

or

$$C_D = 0.496 + 1.33\ (\theta)$$

Reference to Table II shows the close agreement between these predicted drag values and the test values.

CONCLUSIONS

A study of theory and experiment concerning the aerodynamic forces about bodies of revolution, regardless of shape, confirmed the classic theory that the forward flow region which results from fluid flow about these bodies does act in conjunction with the body, and approximates an ellipsoidal shape which is consistant with potential flow theory.

This study revealed definite patterns of relationships between the lift and drag coefficients for the same fineness ratios of the bluff, cone, and hemisphere face bodies of revolution, and the oblate spheroid when these bodies were at varying angles of attack. A further analysis of these patterns was curtailed by the lack of more experimental data for nose configurations of varying degrees of cone angle, and different radii of nose contour curvature.

The analytical prediction for the coefficient of normal lift force was accurate within ten percent of the observed test data for angles of attack that varied from zero to nine degrees. The predicted values have indicated a tendency to converge upon the test data as though this solution is very similar to some form of a series function, and further investigation should be made for a possible series type solution as a method of analytical prediction.

Detailed experimental data of the aerodynamic forces for these bodies is needed in the fineness ratio range < 2, where some body of revolution configurations begin to approach a flat plate and a sphere.

Flow pattern data for the various bodies of revolution must be supplemented so as to realize a more complete appreciation of the similitude of relations that exist between all bodies of revolution when their respective flow patterns are compared with the flow about prolate spheroids.

The drag force prediction method should be investigated through a range of greater than the nine degrees angle of attack which test data made available. Possibly a more complete picture of the C_D vs. angle of attack would permit a curve fitting equation to be applied rather than the straight line slope method used in this solution. In this angle of attack range, the analytical predictions were within ten percent of the test drag coefficients.

In general, the classic potential flow theory will not lead to a solution of the flow characteristics about bodies of revolution unless modifications are utilized that will eliminate the problem of infinite velocities about sharp-edged contours. The procedure for predicting aerodynamic coefficients of drag and lift developed herein will expedite preparation of reasonably accurate predicted aerodynamic design data.

This investigation was limited to the subsonic velocity range, and extensive research should be conducted for a method which would lead to an analytical prediction of the drag and lift forces about bodies of revolution in the supersonic velocity range.

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TABLE I

COEFFICIENT OF DRAG VS. ANGLE OF ATTACK CHARACTERISTICS FOR BODIES OF REVOLUTION

	Bluff Face	Cone Face	Hemisphere Face
For $\ell/d = 2$			
$dC_D/d\theta]_{\ell/d}$	1.02	1.02	1.02
For $\ell/d = 4$			
$dC_D/d\theta]_{\ell/d}$	.99	.99	.99
For $\ell/d = 6$			
$dC_D/d\theta]_{\ell/d}$	1.33	1.33	1.33
For $\ell/d = 8$			
$dC_D/d\theta]_{\ell/d}$	1.30	1.30	1.30
For $\ell/d = 10$			
$dC_D/d\theta]_{\ell/d}$	1.60	1.60	1.60
For Prolate Spheroids (All ℓ/d values > 2)			
$dC_D/d\theta$	.80		

TABLE II

COMPARISON OF PREDICTED AND

MEASURED LIFT AND DRAG COEFFICIENTS

(1)	(2)	(3)	(4)	(5)
$\pm\theta$	Cos θ	Sin θ	C_L (Test Data)	C_D (Test Data)
0	1.000	.000	.000	.494
3	.999	.052	.127	.546
6	.995	.105	.262	.624
9	.988	.156	.381	.706

(6)	(7)	(8)	(9)	(10)
$\pm\theta$	$C_L \text{Cos}\,\theta$	$C_D \text{Sin}\,\theta$	$C_{N_{Test}}$ (7)+(8)	$C_{N_{Predict}}$
0	.000	.000	.00	.00
3	.127	.029	.16	.16
6	.261	.065	.33	.32
9	.376	.110	.48	.48

(11)	(12)	(13)
$\pm\theta$	C_D (Predict)	C_D (Test)
0	.49	.49
3	.56	.54
6	.63	.62
9	.70	.70

FIGURE 1

LAMB'S PROLATE SPHEROID INERTIA COEFFICIENT PRODUCTS

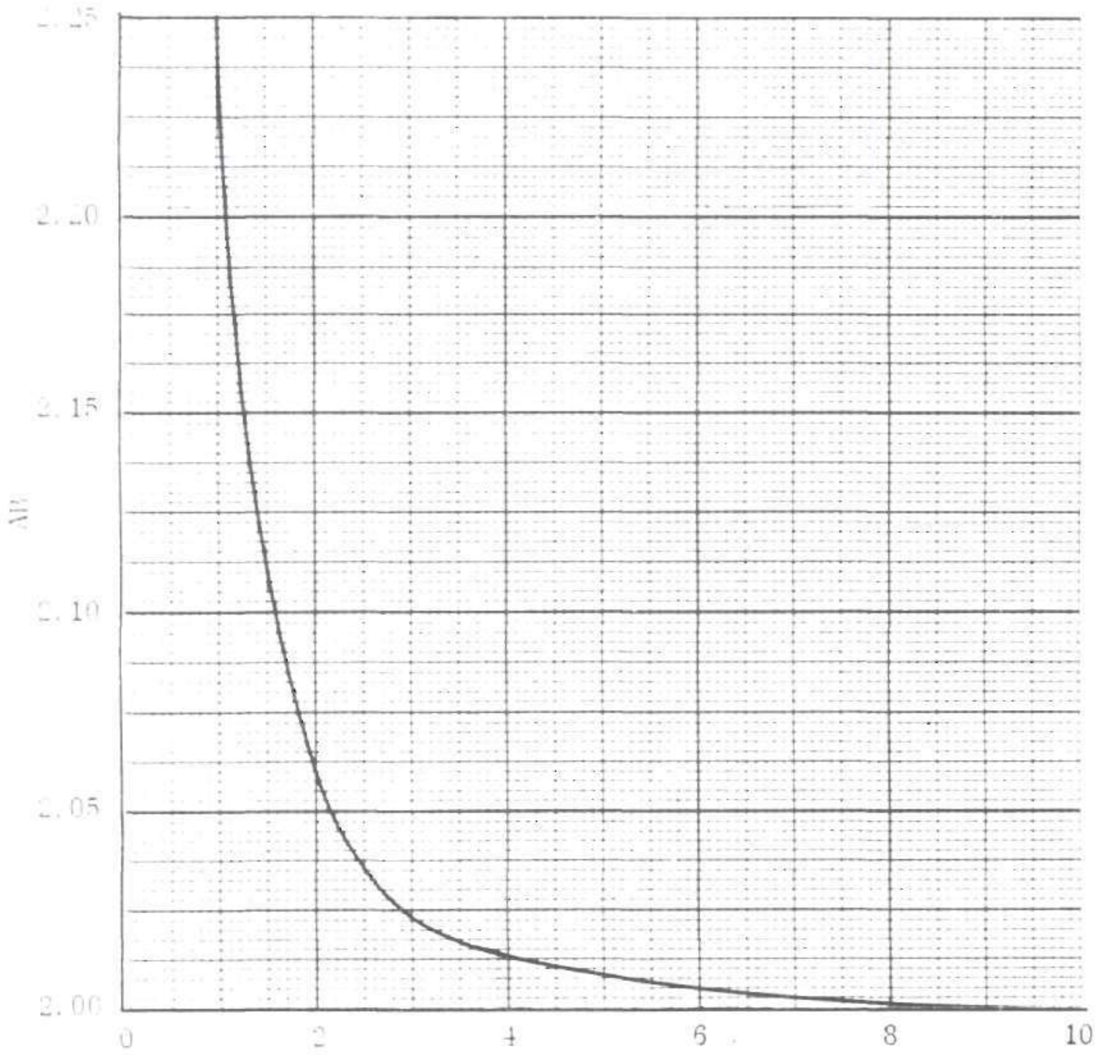


FIGURE 2

REAL FLOW PROLATE SPHEROID EQUIVALENT CURVE

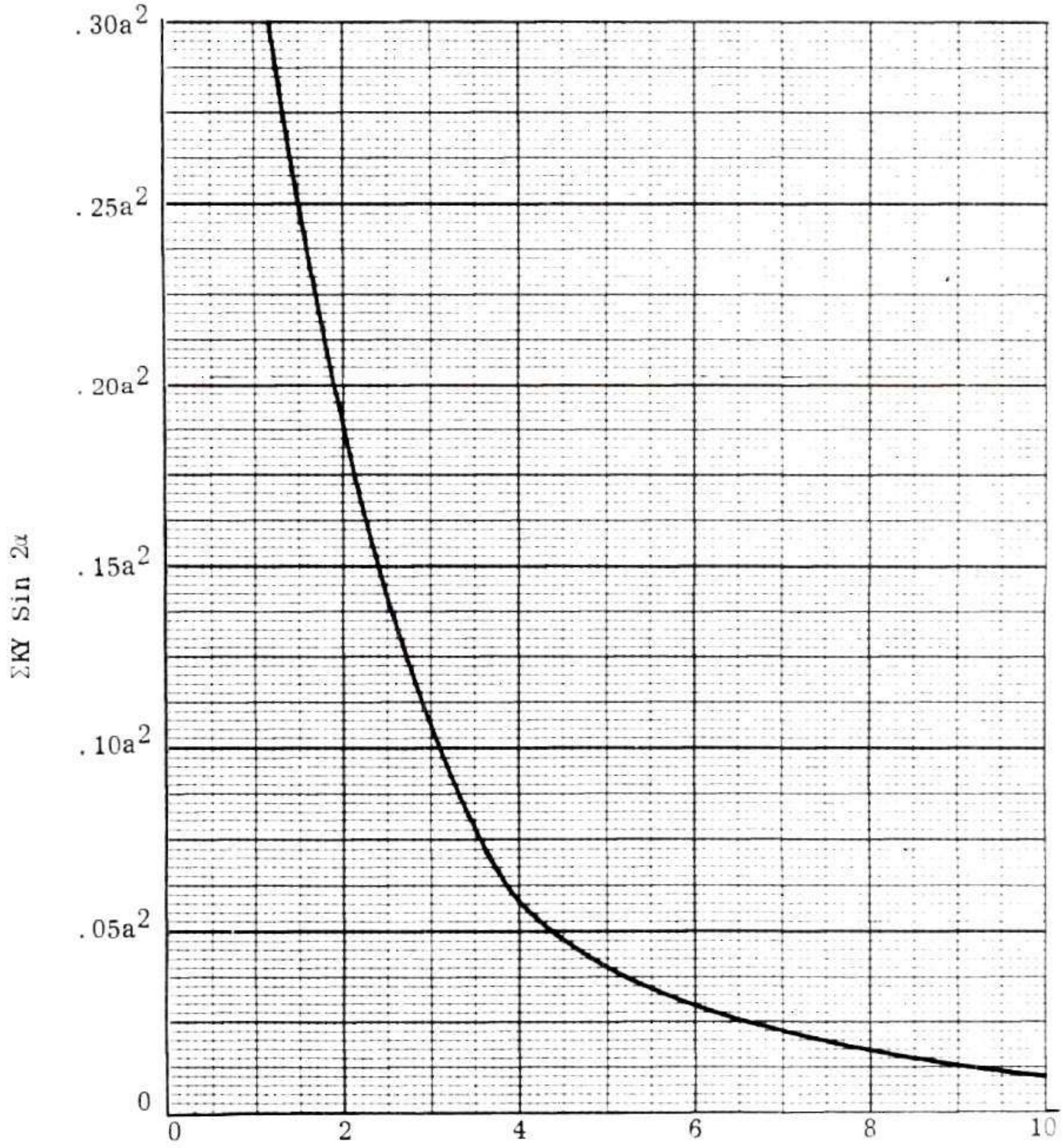


FIGURE 3

INERTIA COEFFICIENTS FOR VARIOUS BODIES OF REVOLUTION

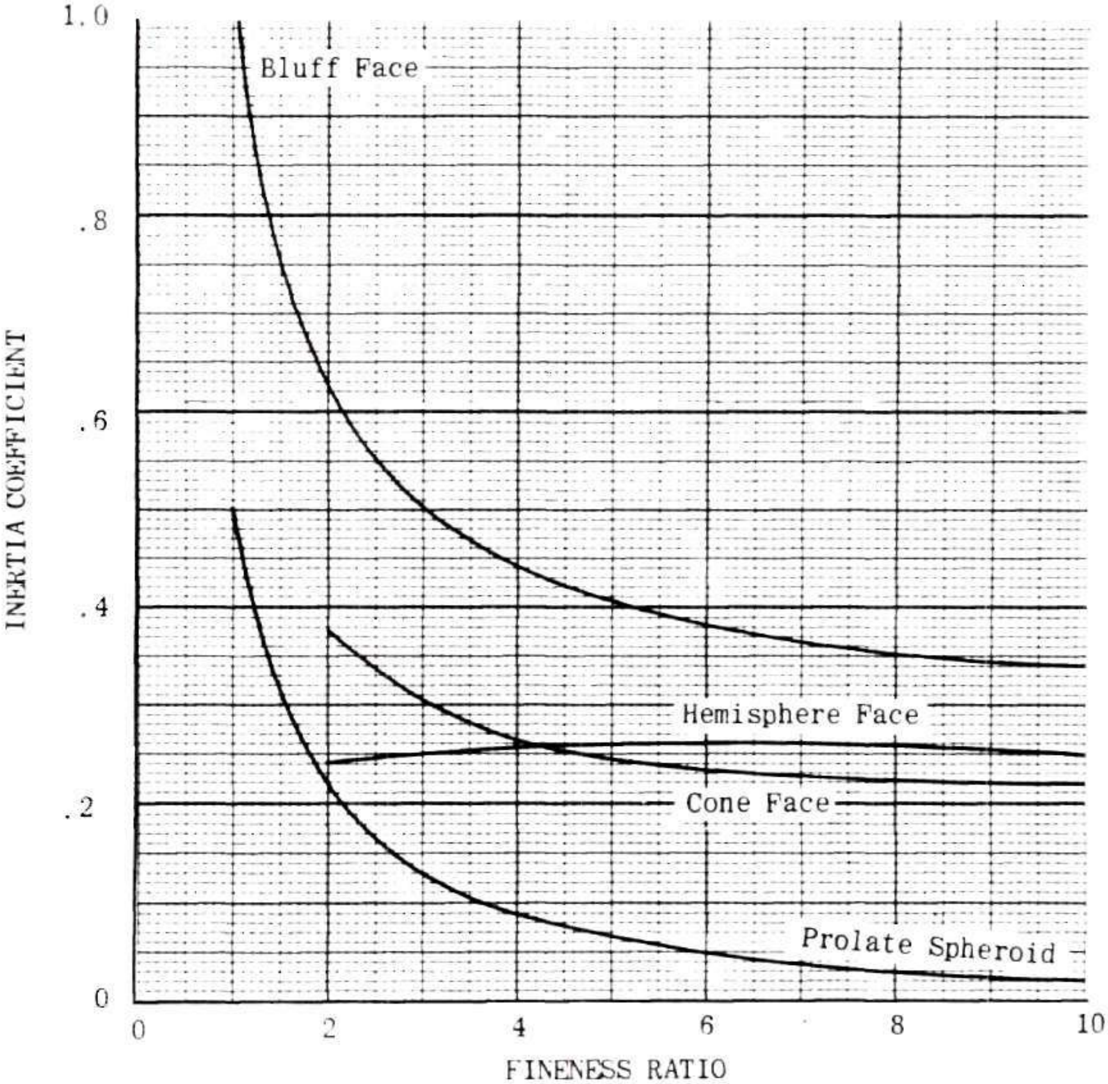


FIGURE 4

CORRECTION RATIO FOR $\Sigma KY \sin 2\alpha$ OF VARIOUS BODIES OR REVOLUTION

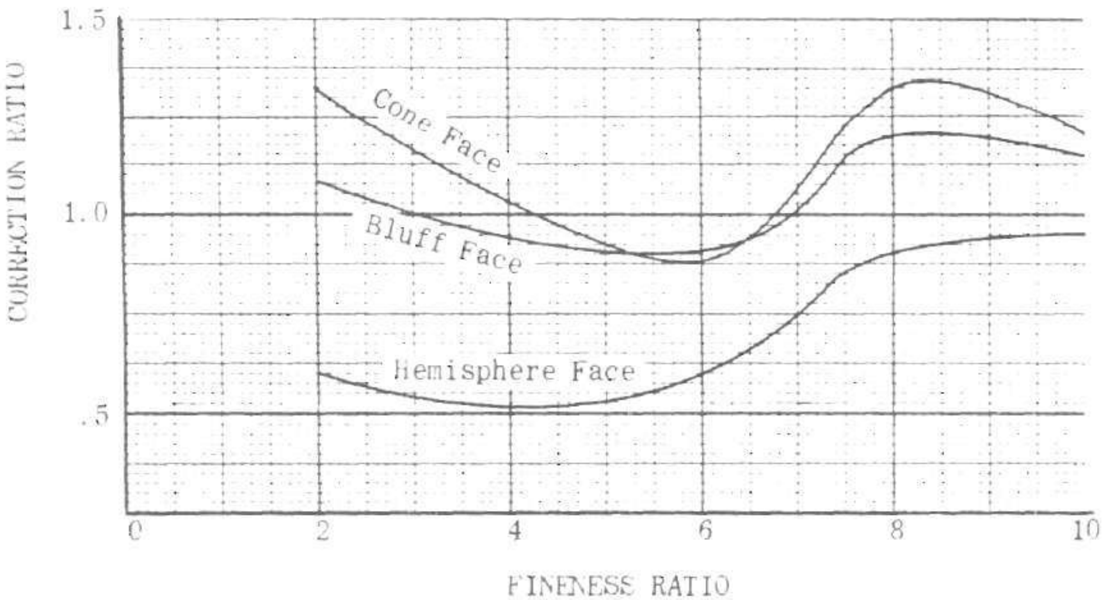


FIGURE 5

COMPARISON OF DRAG COEFFICIENTS AT ZERO ANGLE OF ATTACK

FOR VARIOUS BODIES OF REVOLUTION

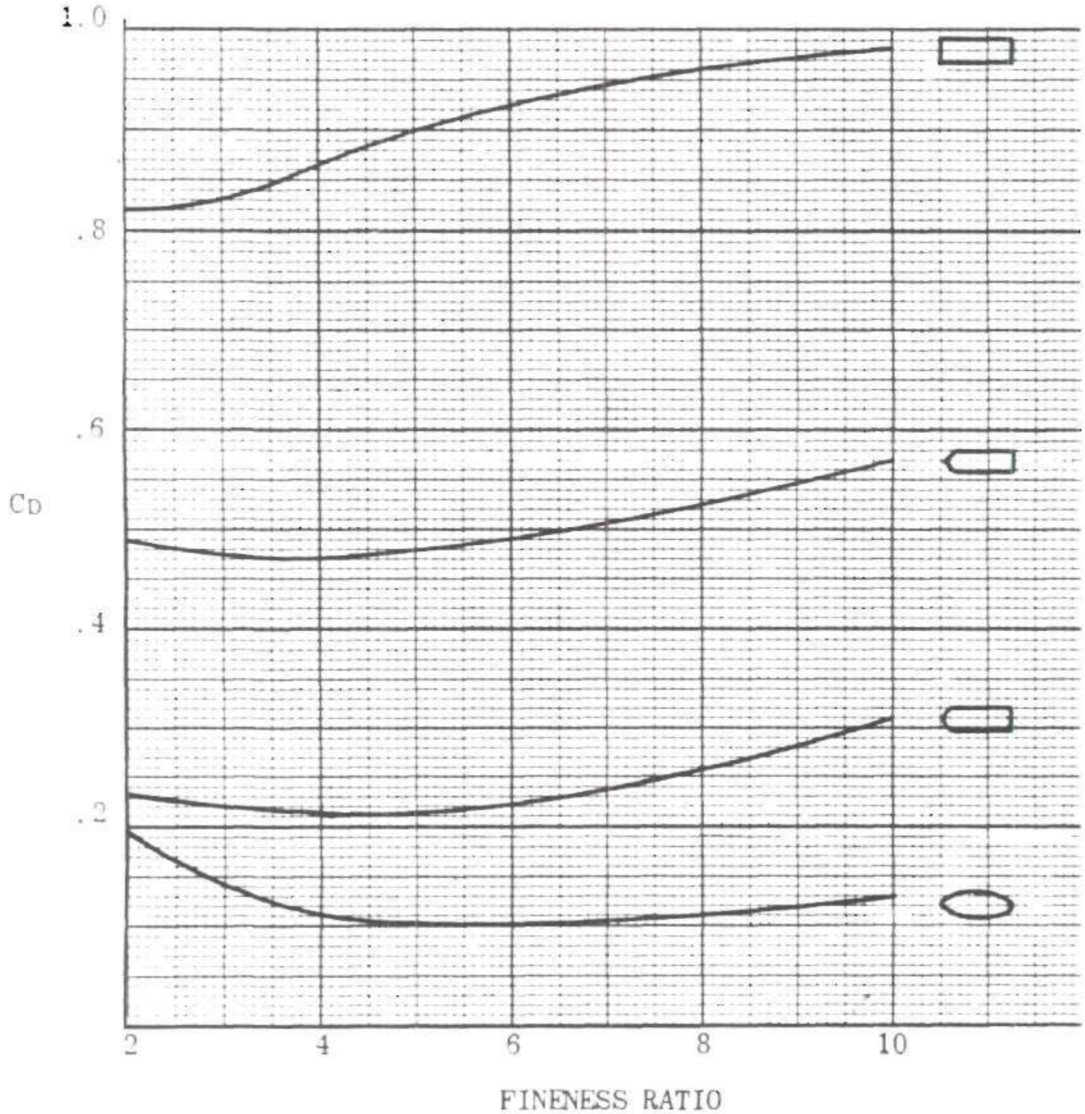


FIGURE 6

NORMAL LIFT COEFFICIENT FOR PROLATE SPHEROID

MAX. DIAM. = 1.5 IN.

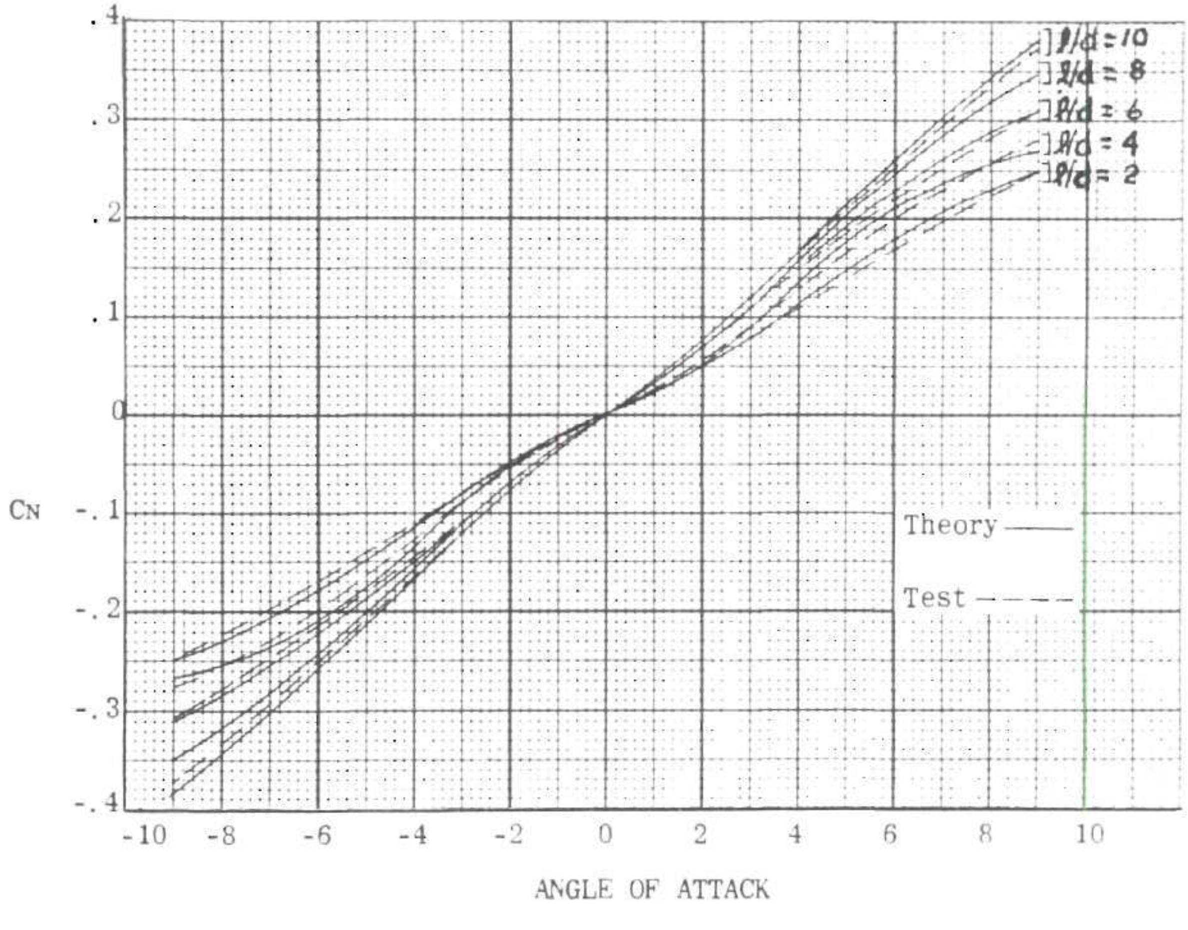


FIGURE 7

NORMAL LIFT COEFFICIENT FOR BLUFF FACE BODY OF REVOLUTION

MAX. DIAM. = 1.5 IN.

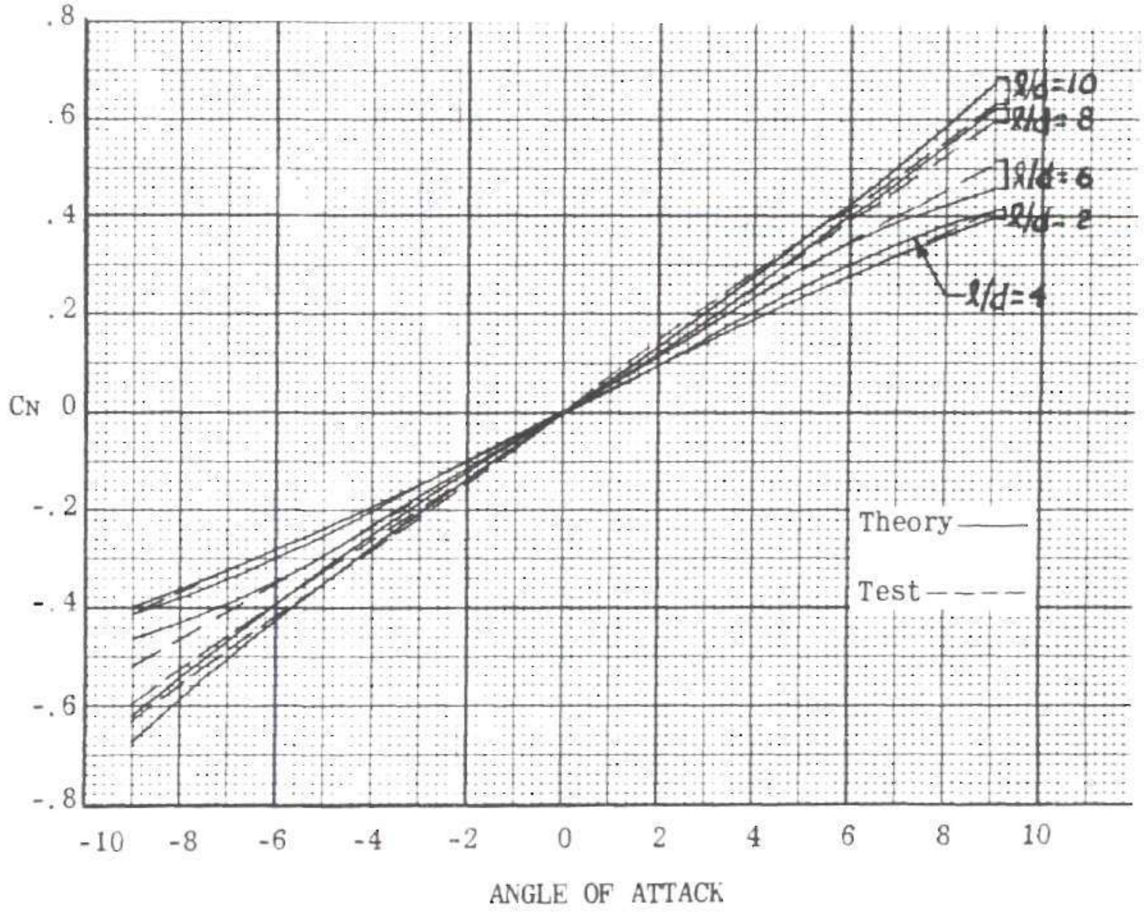


FIGURE 8

NORMAL LIFT COEFFICIENT FOR HEMISPHERE FACE BODY OF REVOLUTION

MAX. DIAM. = 1.5 IN.

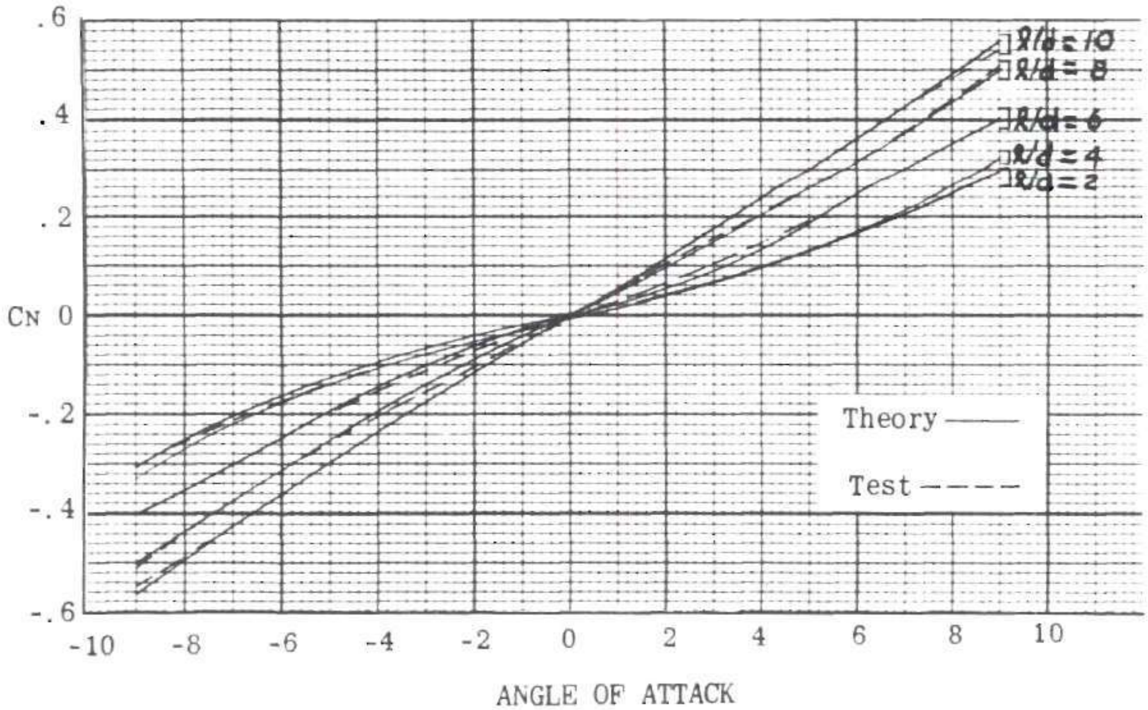


FIGURE 9

NORMAL LIFT COEFFICIENT FOR CONE FACE BODY OF REVOLUTION

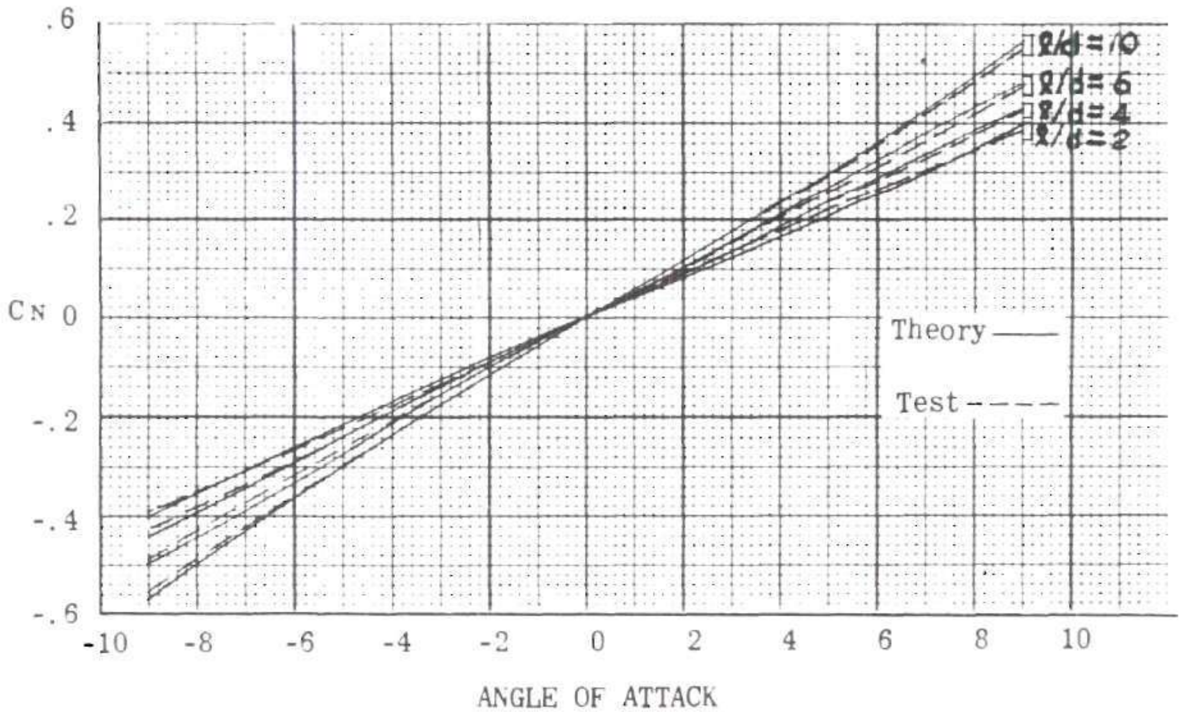


FIGURE 10

DRAG COEFFICIENTS FOR BODIES OF REVOLUTION

MAX. DIAM. = 1.5 IN.

$l/d = 2$

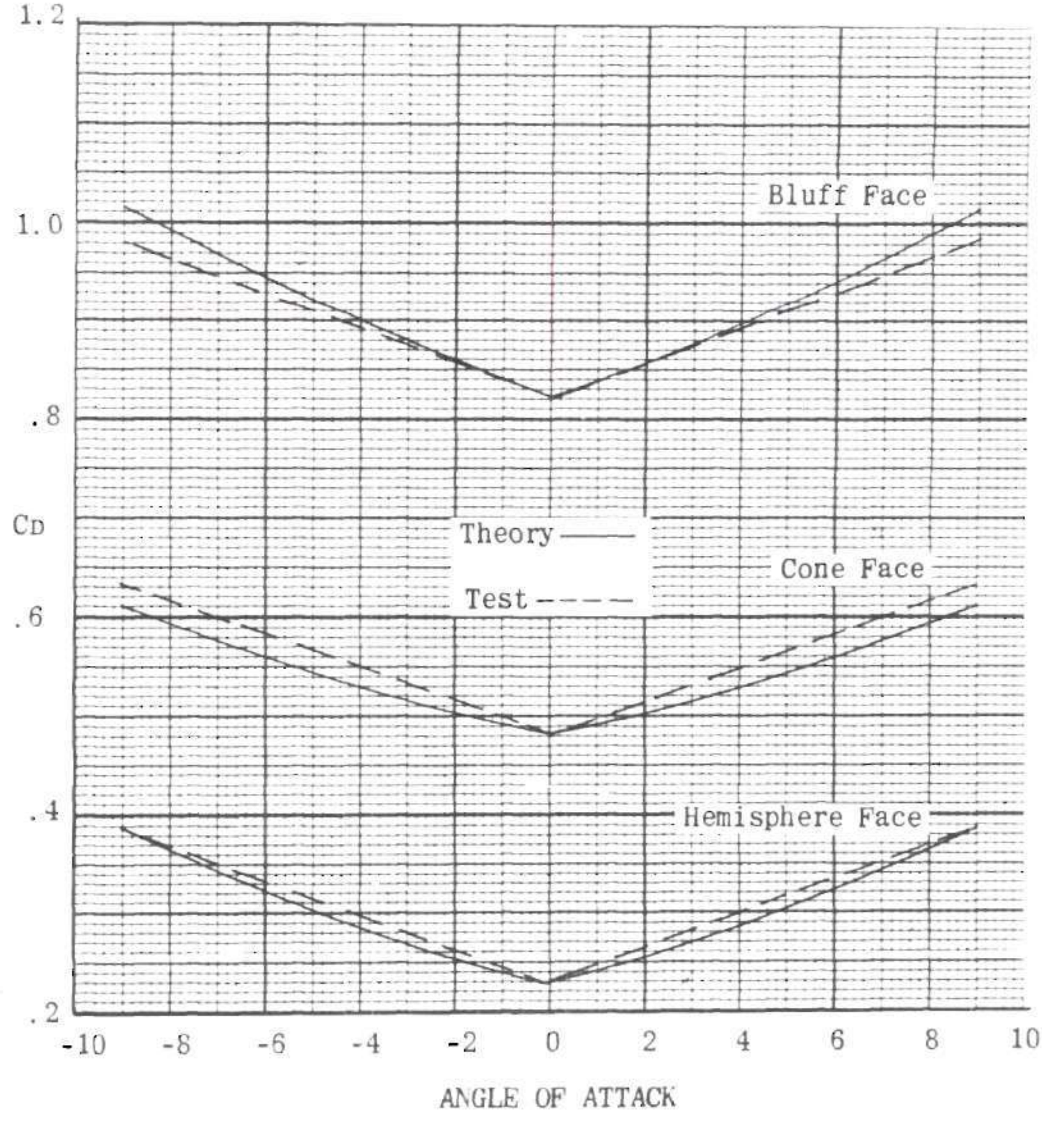


FIGURE 11

DRAG COEFFICIENT FOR BODIES OF REVOLUTION

MAX. DIAM. = 1.5 IN.

l/d = 4

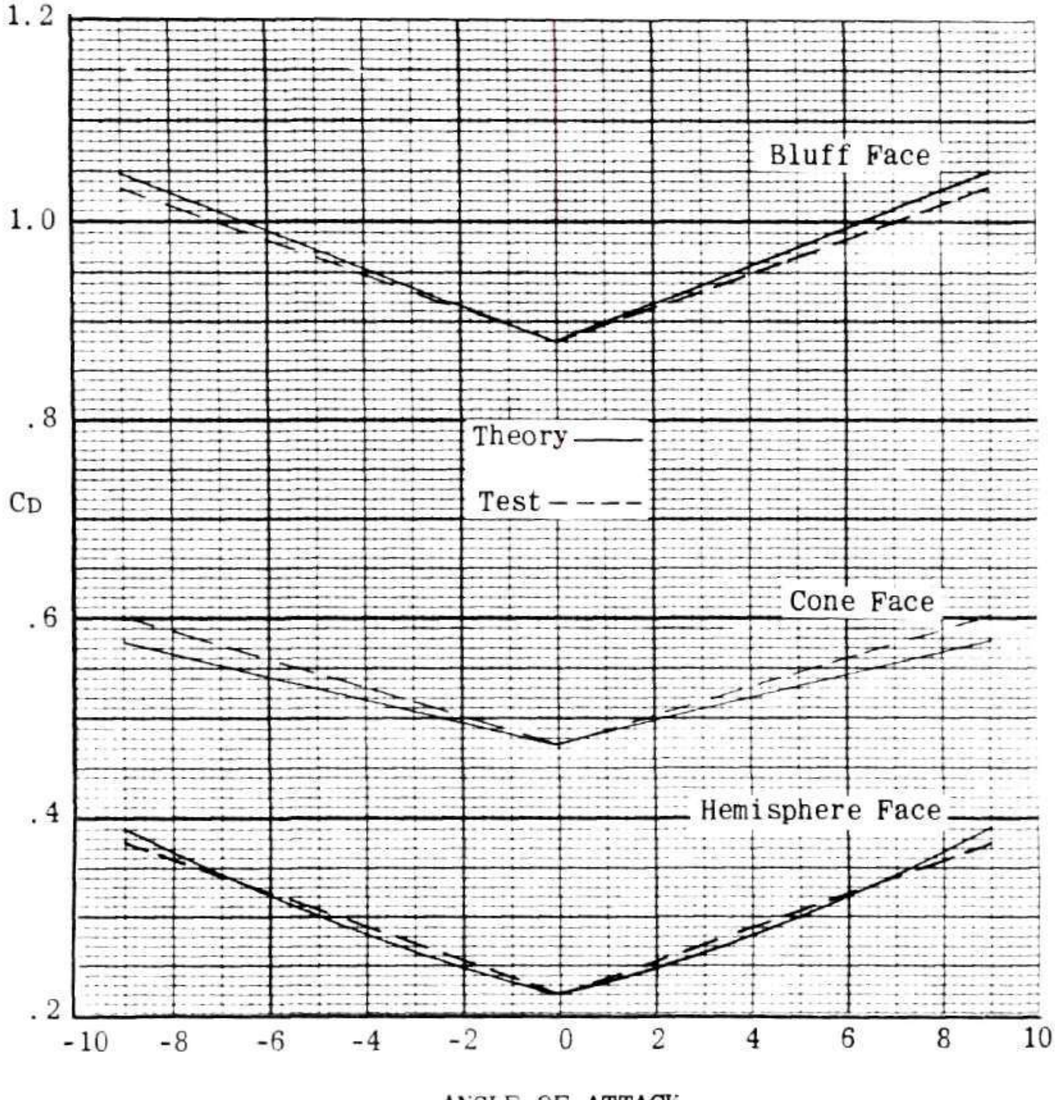


FIGURE 12

DRAG COEFFICIENT FOR BODIES OF REVOLUTION

MAX. DIAM. = 1.5 IN.

l/d = 6

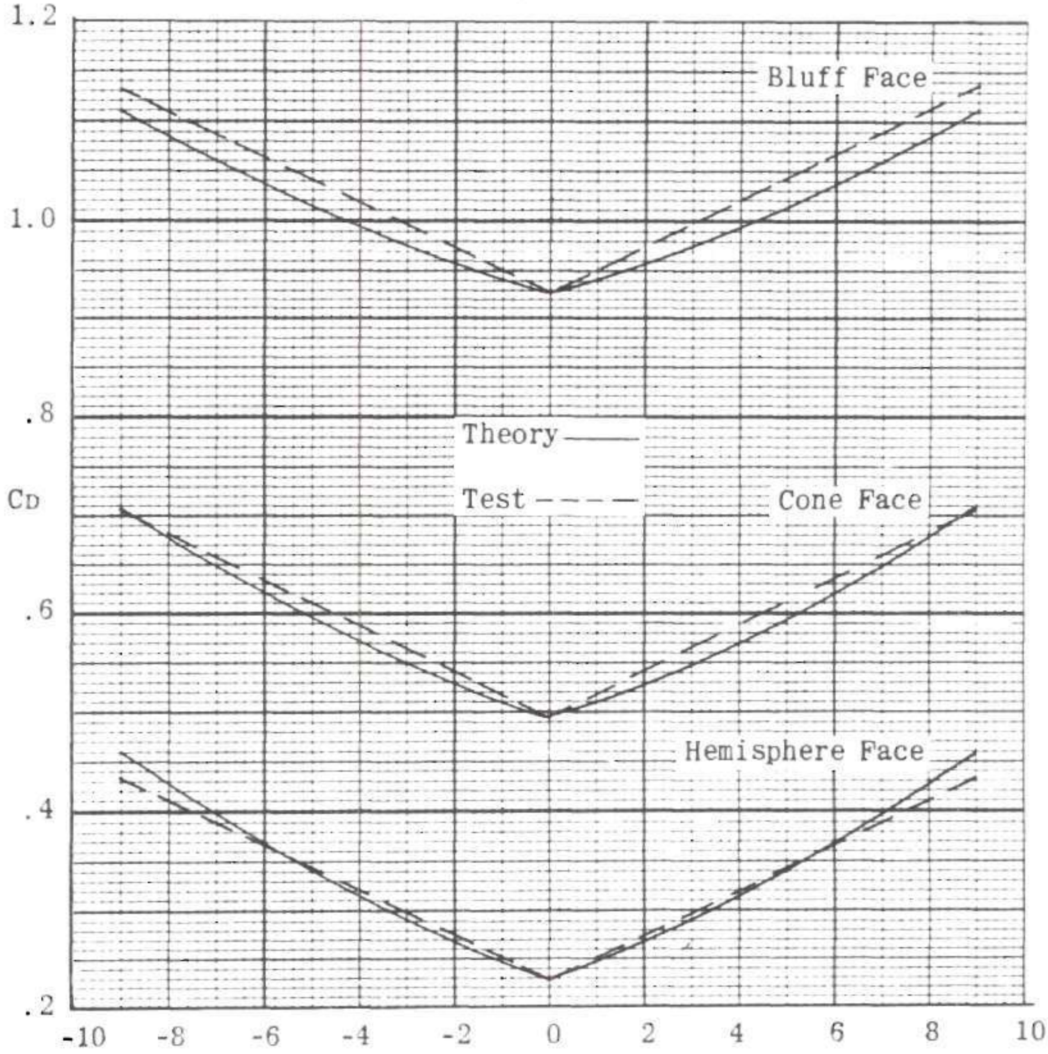


FIGURE 13

DRAG COEFFICIENTS FOR BODIES OF REVOLUTION

MAX. DIAM. = 1.5 IN.

$l/d = 8$

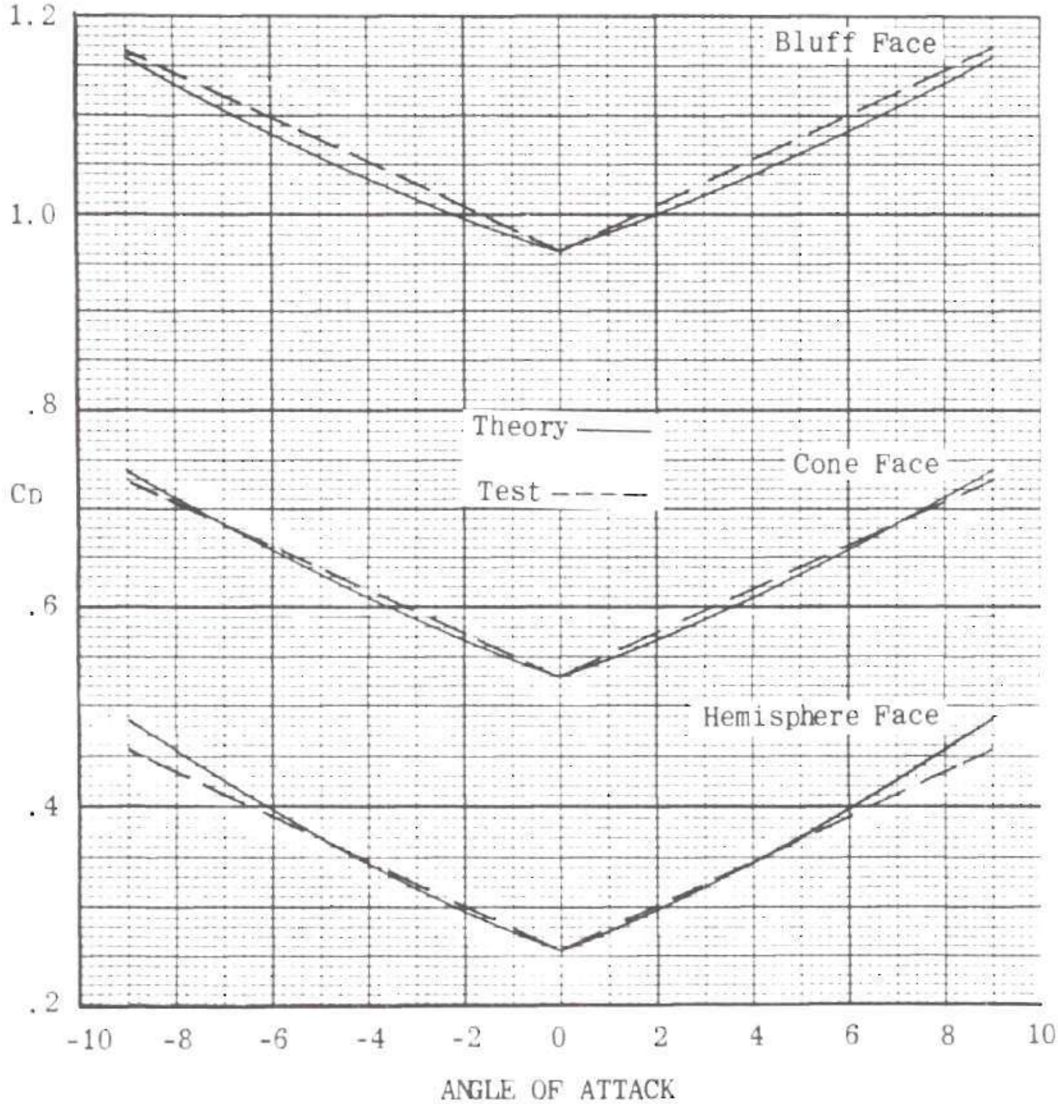


FIGURE 14

DRAG COEFFICIENTS FOR BODIES OF REVOLUTION

MAX. DIAM. = 1.5 IN.

1/d = 10

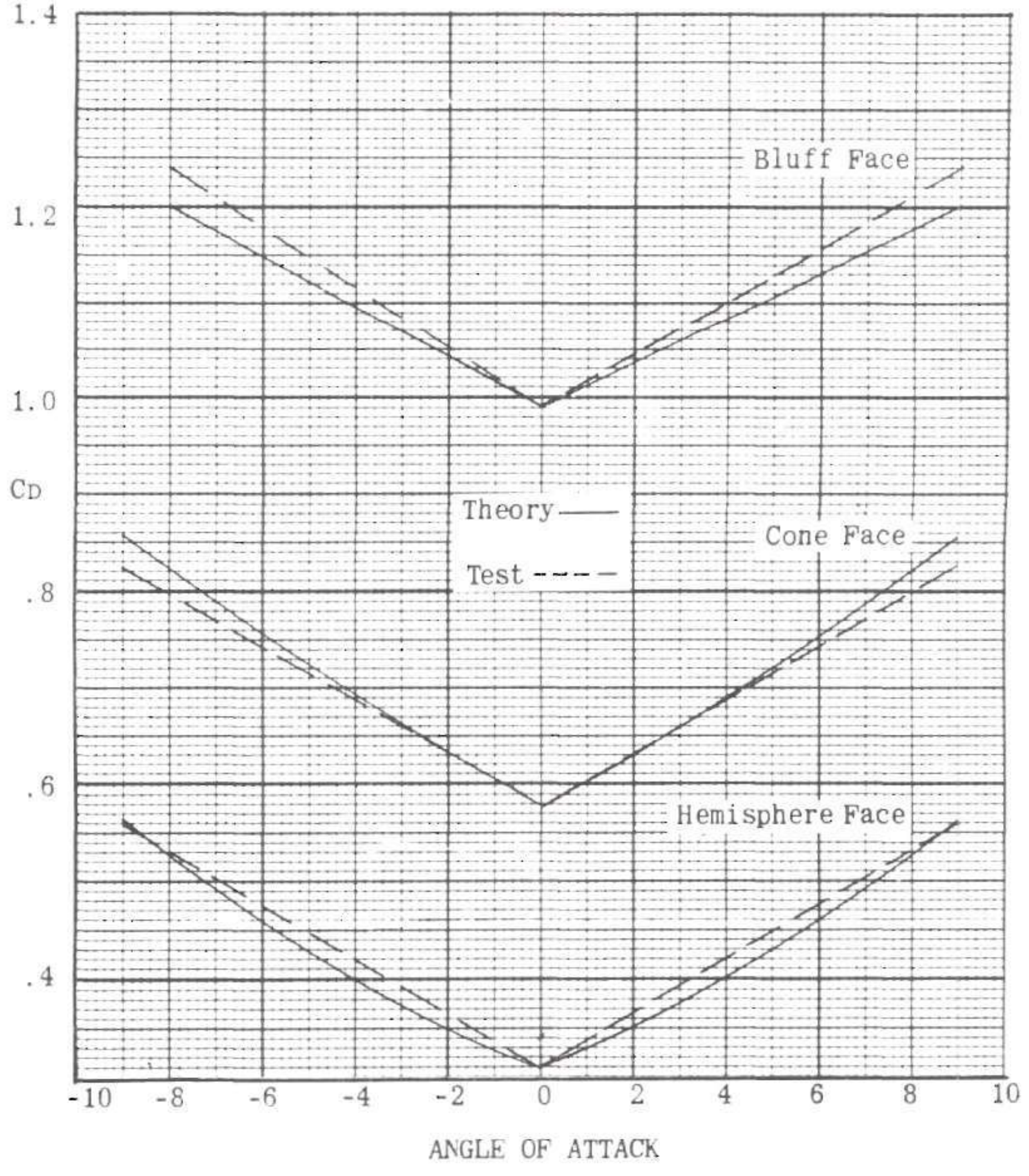


FIGURE 15

DRAG COEFFICIENT FOR PROLATE SPHEROID

MAX. DIAM. = 1.5 IN.

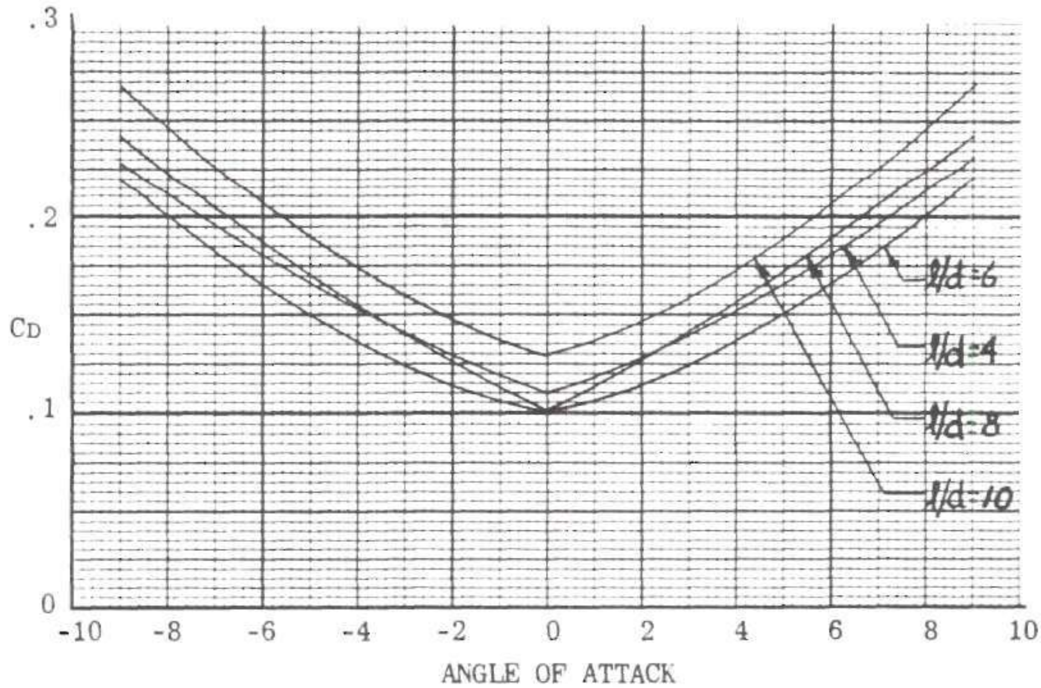


FIGURE 16

LAMB'S INERTIA COEFFICIENTS FOR PROLATE SPHEROID

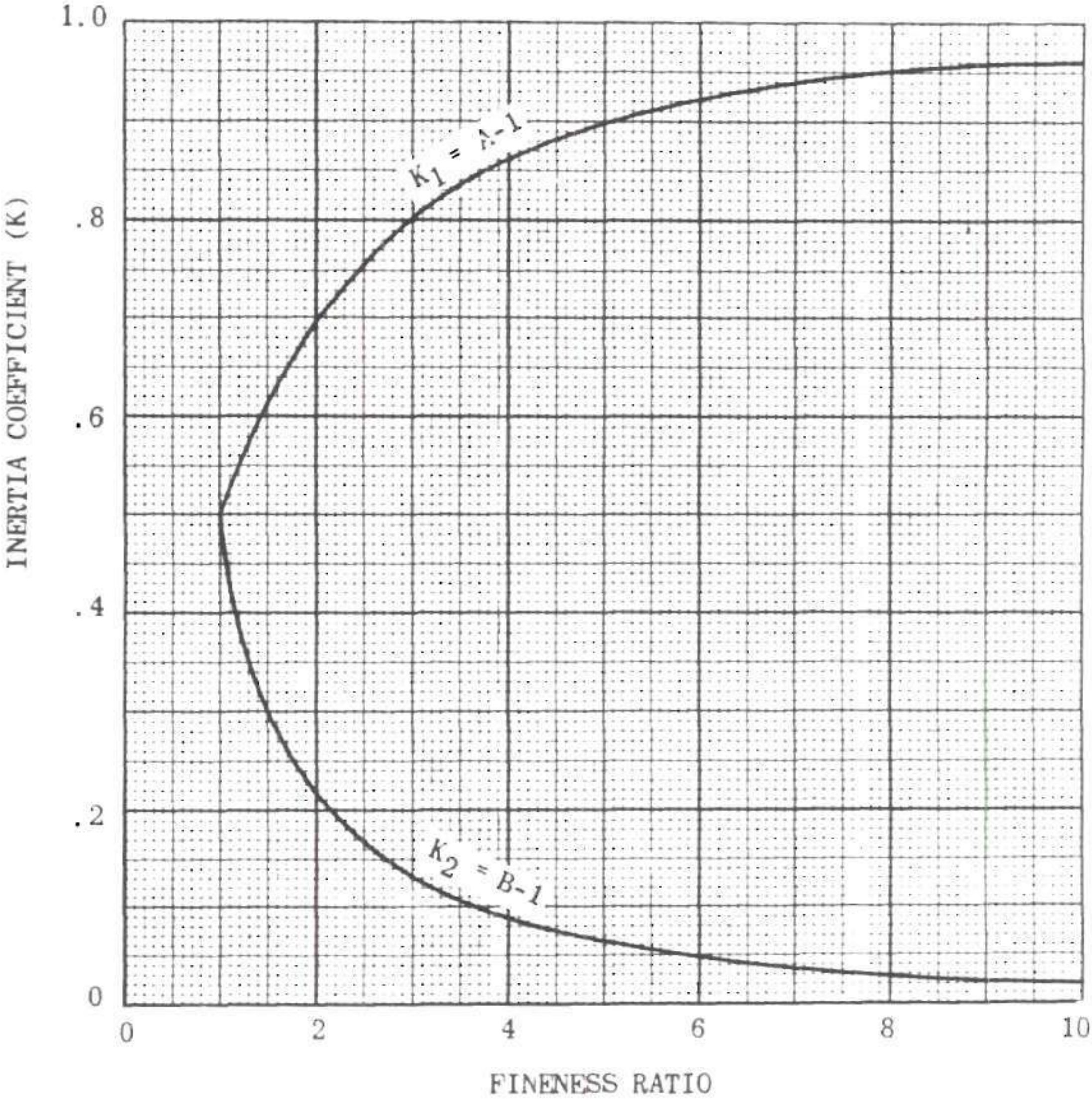
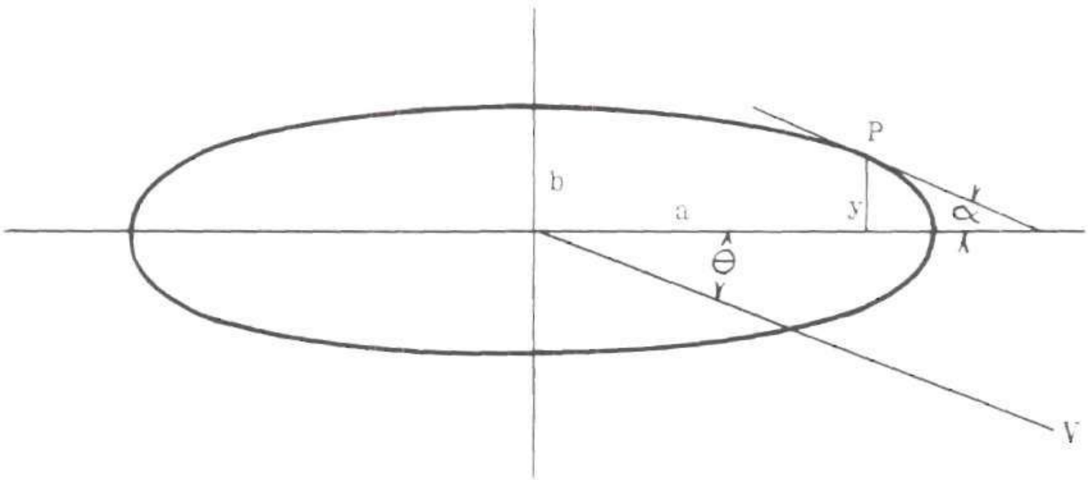


FIGURE 17

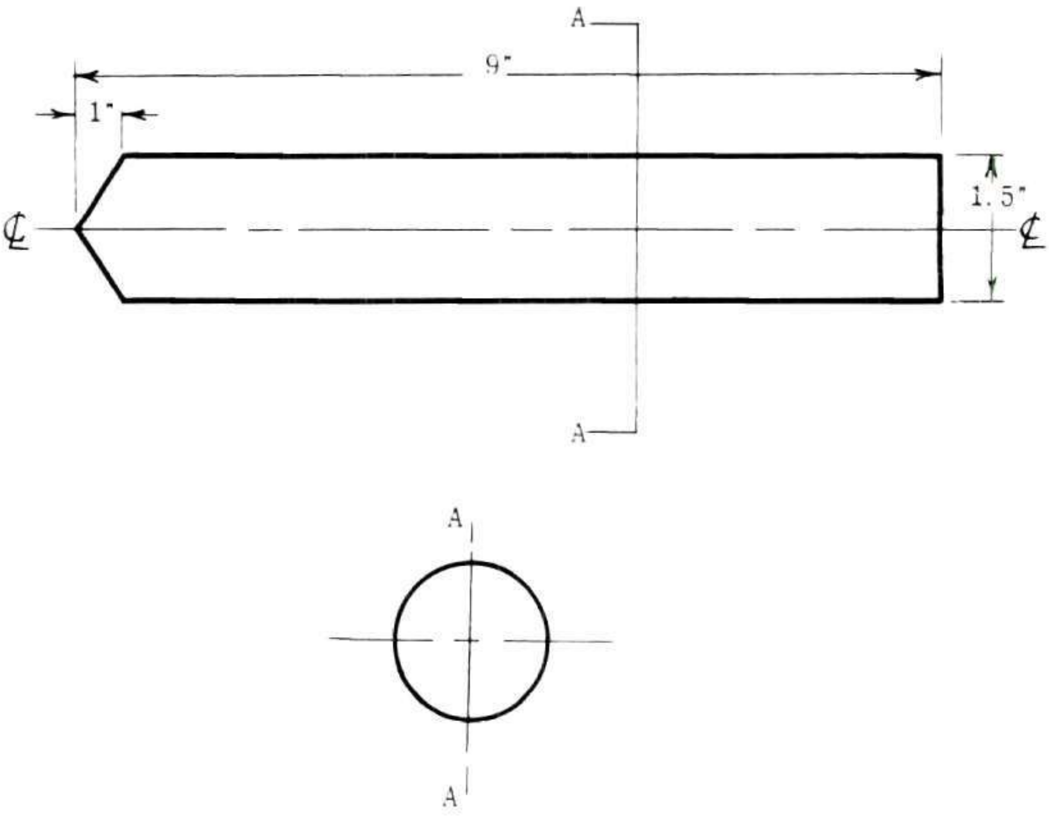


PROLATE SPHEROID SYMBOLS

FOR

BASIC THEORY

FIGURE 18



BODY OF REVOLUTION

FOR

PROCEDURE DEMONSTRATION